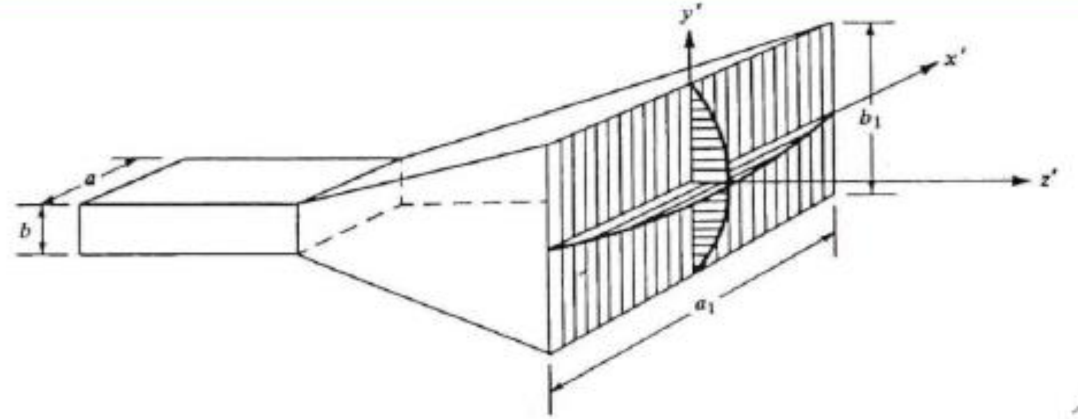
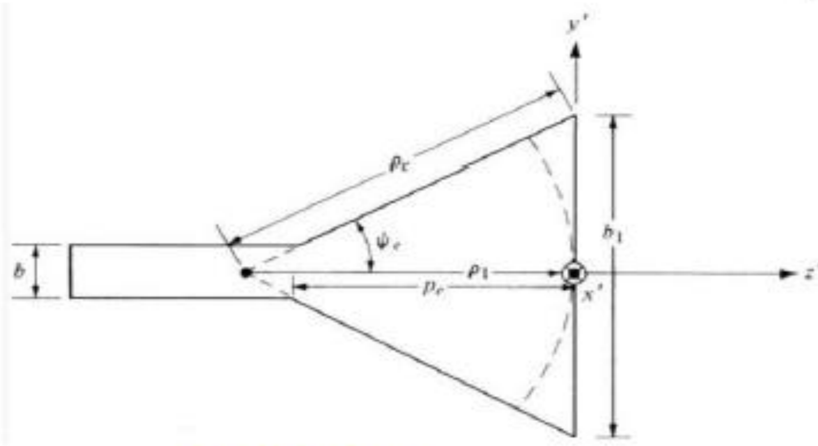
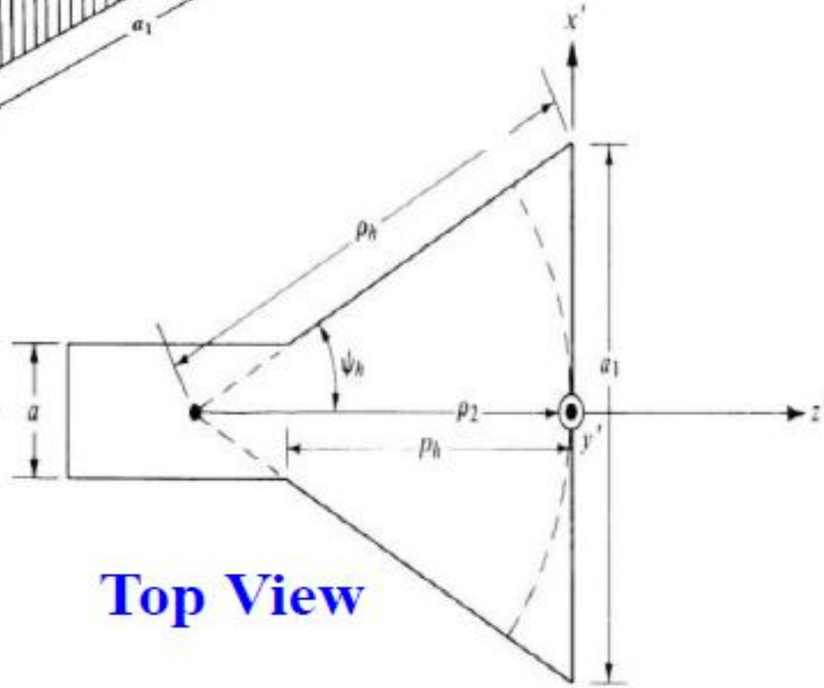
PYRAMIDALHorn

Pyramidal Horn Antenna





Side View



Top View

Pyramidal Horn

Aperture Fields:

$$E'_y(x',y') = E_0 \cos\left(\frac{\pi}{a_1}x'\right) e^{-j\left[k\left(\frac{x'^2}{2\rho_2}+\frac{y'^2}{2\rho_1}\right)\right]} \quad (13\text{-}43a)$$

$$H'_x(x',y') = -\frac{E_0}{\eta} \cos\left(\frac{\pi}{a_1}x'\right) e^{-j\left[k\left(\frac{x'^2}{2\rho_2}+\frac{y'^2}{2\rho_1}\right)\right]} \quad (13\text{-}43b)$$

Radiated Fields:

$$E_\theta = j\frac{kE_0 e^{-jkr}}{4\pi r}\left[\sin\phi(1+\cos\theta)I_1 I_2\right] \quad (13\text{-}48b)$$

$$E_\phi = j\frac{kE_0 e^{-jkr}}{4\pi r}\left[\cos\phi(1+\cos\theta)I_1 I_2\right] \quad (13\text{-}48c)$$

$$I_1 = \frac{1}{2}\sqrt{\frac{\pi\rho_2}{k}}\ \{e^{j\frac{k_x'^2\rho_2}{2k}}\left[\left(C(t_2')-C(t_1')\right)-j\left(S(t_2')-S(t_1')\right)\right]$$

$$+e^{j\frac{k_x''^2\rho_2}{2k}}\left[\left(C(t_2'')-C(t_1'')\right)-j\left(S(t_2'')-S(t_1'')\right)\right]\} \quad (13\text{-}46)$$

$$I_2 = \sqrt{\frac{\pi\rho_1}{k}}e^{j\frac{k_y^2\rho_1}{2k}}\left\{\left[\left(C(t_2)-C(t_1)\right)-j\left(S(t_2)-S(t_1)\right)\right]\right\} \quad (13\text{-}47)$$

$$\rho_1 = \rho_2 = 6\lambda$$

$$a_1 = 5.5\lambda$$

$$b_1 = 2.75\lambda$$

$$a = 0.5\lambda$$

$$b = 0.25\lambda$$

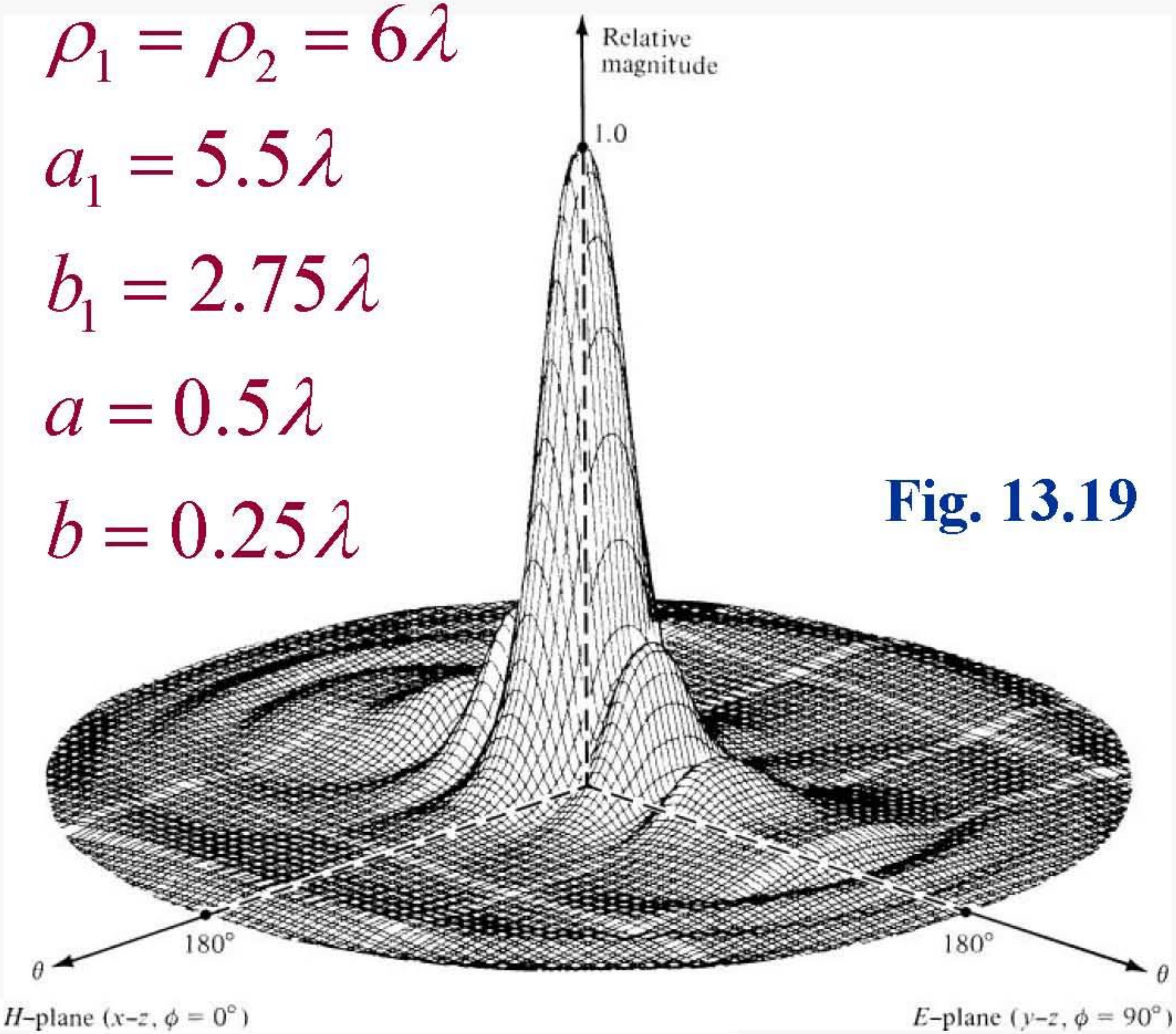


Fig. 13.19

$\rho_1 = \rho_2 = 6\lambda$

$a_1 = 12\lambda$

$b_1 = 6\lambda$

$a = 0.5\lambda$

$b = 0.25\lambda$

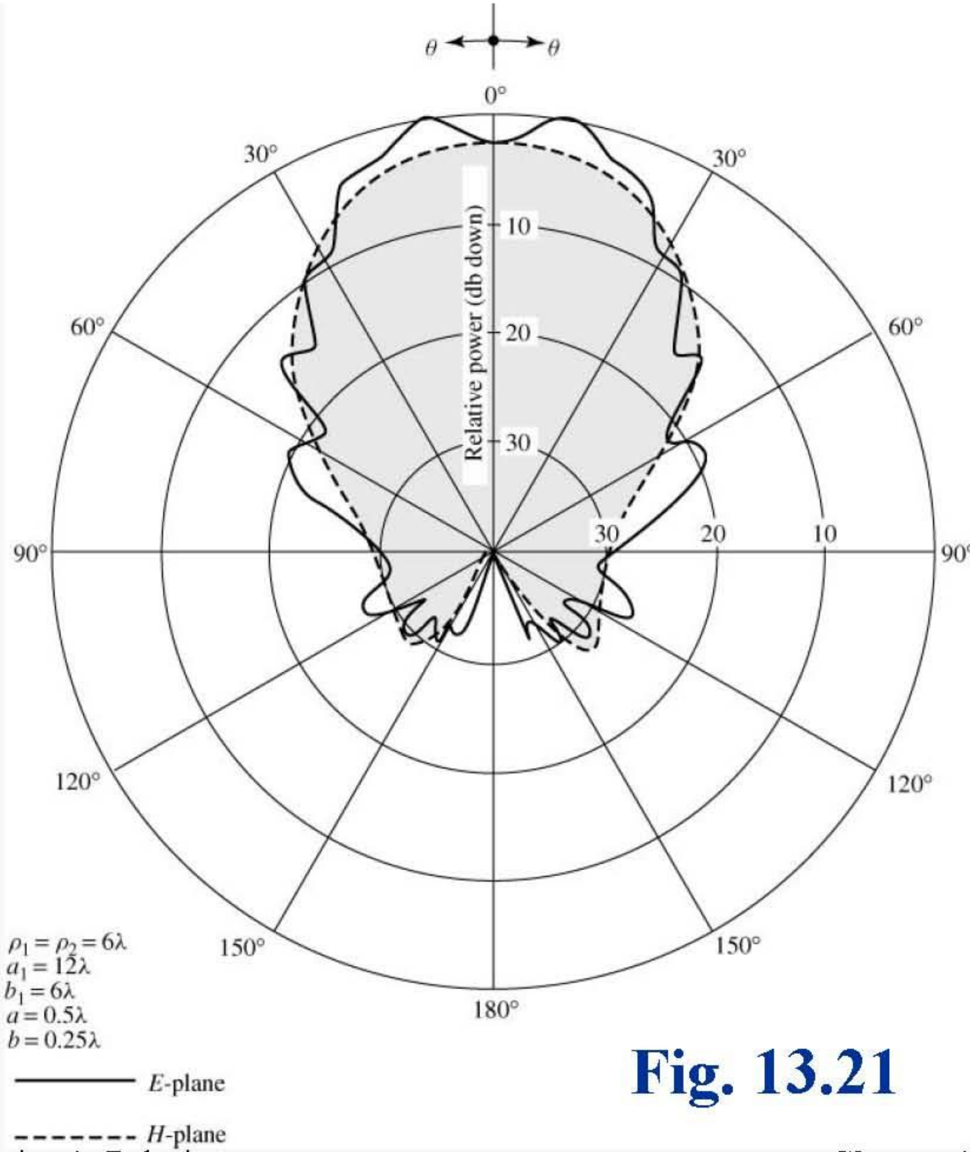


Fig. 13.21

Pyramidal Horn Antenna

$$E'_y(x', y') = E_0 \cos\left(\frac{\pi}{a_1} x'\right) e^{-j\left[k\left(\frac{x'^2}{2\rho_2} + \frac{y'^2}{2\rho_1}\right)\right]}$$

Condition for Physical Realization:

$$p_e = (b_1 - b)\left[\left(\frac{\rho_e}{b_1}\right)^2 - \frac{1}{4}\right]^{1/2}$$

$$p_h = (a_1 - a)\left[\left(\frac{\rho_h}{a_1}\right)^2 - \frac{1}{4}\right]^{1/2}$$

$$\boxed{p_e = p_h}$$

Pyramidal Horn: Design Procedure

Directivity of Pyramidal Horn Antenna can be obtained using Directivity curves for E-and H-Planes Sectoral Horn antenna

$$D_p = \frac{\pi\lambda^2}{32ab} D_E D_H$$

Alternatively

$$G_0 \simeq \frac{1}{2}\left(\frac{4\pi}{\lambda^2} a_1 b_1 \right)$$

$$a_1 \simeq \sqrt{3\lambda\rho_2} \approx \sqrt{3\lambda\rho_h} \qquad \rho_2 \simeq \rho_h$$

$$b_1 \simeq \sqrt{2\lambda\rho_1} \approx \sqrt{2\lambda\rho_e} \qquad \rho_1 \simeq \rho_e$$

$$p_e = (b_1 - b)\sqrt{\left(\frac{p_e}{b_1}\right)^2 - \frac{1}{4}}$$

$$p_h = (a_1 - a)\sqrt{\left(\frac{p_h}{a_1}\right)^2 - \frac{1}{4}}$$

Examples

1. $\rho_1 = \rho_2 = 6\lambda,\ a_1 = 5.5\lambda,\ b_1 = 2.75\lambda,\ a = 0.5\lambda,\ b = 0.25\lambda$

$$\rho_e = \sqrt{\rho_1^{\,2} + (b_1/2)^2} = \sqrt{(6)^2 + (2.75/2)^2}\,\lambda = 6.1555\lambda$$

$$\rho_h = \sqrt{\rho_2^{\,2} + (a_1/2)^2} = \sqrt{(6)^2 + (5.5/2)^2}\,\lambda = 6.6\lambda$$

$$p_e = (2.75 - 0.25)\lambda = \left[\left(\frac{6.1555}{2.75}\right)^2 - \frac{1}{4}\right]^{1/2} = 5.4544\lambda$$

$$p_h = (5.5 - 0.5)\lambda = \left[\left(\frac{6.6}{5.5}\right)^2 - \frac{1}{4}\right]^{1/2} = 5.4544\lambda$$

$$p_e = p_h = 5.4544\lambda$$

$$2.\ \rho_1 = \rho_2 = 6\lambda,\ a_1 = 12\lambda,\ b_1 = 6\lambda,\ a = 0.5\lambda,\ b = 0.25\lambda$$

$$\rho_e = \sqrt{\rho_1^{\,2} + (b_1/2)^2} = \sqrt{(6)^2 + (6/2)^2}\,\lambda = 6.7082\lambda$$

$$\rho_h = \sqrt{\rho_2^{\,2} + (a_1/2)^2} = \sqrt{(6)^2 + (12/2)^2}\,\lambda = 8.4853\lambda$$

$$p_e = (6 - 0.25)\lambda \quad = \left[\left(\frac{6.7082}{6}\right)^2 - \frac{1}{4}\right]^{1/2} \quad = 5.75\lambda$$

$$p_h = (12 - 0.5)\lambda \quad = \left[\left(\frac{8.4853}{12}\right)^2 - \frac{1}{4}\right]^{1/2} \quad = 5.75\lambda$$

$$p_e = p_h = 5.75\lambda$$

Design of Pyramidal horn antenna

Design Procedure

- *To design a pyramidal horn, one usually knows the desired gain G0 and the dimensions a, b of the rectangular feed waveguide.*

- *The objective of the design is to determine the remaining dimensions (a1, b1, ρe, ρh, Pe, and Ph) that will lead to an optimum gain.*

The design equations are derived by firsts electing values of a1 and b1that lead to optimum directivities for the E- and H-plane sectoral horns.

$$a_1 \simeq \sqrt{3\lambda\rho_2}$$

$$b_1 \simeq \sqrt{2\lambda\rho_1}$$

Since the overall efficiency (including both the antenna and aperture efficiencies) of a horn antenna is about 50% .
the gain of the antenna can be related to its physical area.
Thus it can be written by

$$G_0 = \frac{1}{2}\frac{4\pi}{\lambda^2}(a_1 b_1) = \frac{2\pi}{\lambda^2}\sqrt{3\lambda\rho_2}\sqrt{2\lambda\rho_1} \simeq \frac{2\pi}{\lambda^2}\sqrt{3\lambda\rho_h}\sqrt{2\lambda\rho_e}$$

since for long horns ρ2 ≈ ρh and ρ1 ≈ ρe.
For a pyramidal horn to be physically realizable, Pe and Ph must be equal. Using this equality, it can be shown that gain reduces to

$$\left(\sqrt{2\chi} - \frac{b}{\lambda}\right)^2 (2\chi - 1) = \left(\frac{G_0}{2\pi}\sqrt{\frac{3}{2\pi}}\frac{1}{\sqrt{\chi}} - \frac{a}{\lambda}\right)^2 \left(\frac{G_0^2}{6\pi^3}\frac{1}{\chi} - 1\right)$$

where

$$\frac{\rho_e}{\lambda} = \chi \tag{13-56a}$$

$$\frac{\rho_h}{\lambda} = \frac{G_0^2}{8\pi^3}\left(\frac{1}{\chi}\right) \tag{13-56b}$$

This Equation is the horn-design equation .

1. *As a first step of the design, find the value of χ which satisfies (13-56) for a desired gain G0 (dimensionless). Use an iterative technique and begin with a trial value of*

$$\chi\,(\text{trial}) = \chi_1 = \frac{G_0}{2\pi\sqrt{2\pi}}$$

2. Once the correct χ has been found, determine ρe and ρh using (13-56a) and (13-56b) respectively.

3. Find the corresponding values of a1 and b1.

$$a_1 = \sqrt{3\lambda\rho_2} \simeq \sqrt{3\lambda\rho_h} = \frac{G_0}{2\pi}\sqrt{\frac{3}{2\pi\,\chi}}\lambda$$

$$b_1 = \sqrt{2\lambda\rho_1} \simeq \sqrt{2\lambda\rho_e} = \sqrt{2\chi\,\lambda}$$

Problems of horn antenna

Example 13.6:

Given: X-Band (8.2-12.4 GHz), f = 11 GHz Horn; Gain=22.6 dB

$a = 0.9$ in (2.286 cm), $b = 0.4$ in (1.016 cm)

Find: Dimensions Of Pyramidal Horn

Solution

$$G_0(dB) = 22.6 = 10\log_{10} G_0 \Rightarrow G_0 = 10^{2.26} = 181.97$$

$$At\ f = 11 \text{ GHz} \Rightarrow \lambda = \frac{30 \text{ x } 10^9}{11 \text{ x } 10^9} = 2.7273\ cm$$

$$b = \frac{1.016}{2.7273}\lambda = 0.3725\lambda;\ a = \frac{2.286}{2.7273}\lambda = 0.8382\lambda$$

1. Initial value of χ

$$\chi_1 = \frac{G_0}{2\pi\sqrt{2\pi}} = \frac{181.97}{2\pi\sqrt{2\pi}} = 11.5539$$

which does not satisfy(12-56), or

$$\left(\sqrt{2\chi} - \frac{b}{\chi}\right)^2 (2\chi - 1) = \left(\frac{G_0}{2\pi}\sqrt{\frac{3}{2\pi}}\frac{1}{\sqrt{\chi}} - \frac{a}{\lambda}\right)^2 \left(\frac{G_0{}^2}{6\pi^3}\frac{1}{\chi} - 1\right)$$

After few tries, a more accurate value is

$$\chi = 11.1157$$

2. $\rho_e = \chi\lambda = 11.1157\lambda = 30.316\ cm = 11.935\ in.$

$$\rho_h = \frac{G_0{}^2}{8\pi^3}\left(\frac{1}{\chi}\right)\lambda = 12.0094\lambda = 32.753\ cm = 12.895\ in.$$

3. $$a_1 = \sqrt{3\lambda\rho_2} \approx \sqrt{3\lambda\rho_h} = \frac{G_0}{2\pi}\sqrt{\frac{3}{2\pi\chi}}\lambda = 6.002\lambda$$

$$= 16.370\ cm = 6.445\ in.$$

$$b_1 = \sqrt{2\lambda\rho_1} \approx \sqrt{2\lambda\rho_e} = \sqrt{2\chi\lambda} = 4.715\lambda$$

$$= 12.859\ cm = 5.063\ in.$$

4. $$p_e = (b_1 - b)\left[\left(\frac{p_e}{b_1}\right)^2 - \frac{1}{4}\right]^{1/2} = 10.005\lambda$$

$$= 27.286\ cm = 10.743\ in.$$

$$p_h = (a_1 - a)\left[\left(\frac{p_h}{a_1}\right)^2 - \frac{1}{4}\right]^{1/2} = 10.005\lambda$$

$$= 27.286\ cm = 10.743\ in.$$

13.21. Design a pyramidal horn antenna with optimum gain at a frequency of 10 GHz. The overall length of the antenna from the imaginary vertex of the horn to the center of the aperture is 10λ and is nearly the same in both planes. Determine the

(a) Aperture dimensions of the horn (*in cm*).

(b) Gain of the antenna (*in dB*)

(c) Aperture efficiency of the antenna (*in %*). Assume the reflection, conduction, and dielectric losses of the antenna are negligible.

(d) Power delivered to a matched load when the incident power density is 10 μwatts/m^2.

13-18. $\lambda = \frac{30 \times 10^9}{10 \times 10^9} = 3\ cm$

a. $a_1 \simeq \sqrt{3\lambda\rho} = \sqrt{3\lambda(10\lambda)} = \sqrt{30\lambda^2} = 5.477\lambda = 16.43\ cm$

$b_1 \simeq \sqrt{2\lambda\rho} = \sqrt{20\lambda^2} = 4.472\lambda = 13.416\ cm$

b. $G_o = \frac{1}{2}\frac{4\pi}{\lambda^2}(a_1 b_1) = \frac{1}{2}\frac{4\pi}{\lambda^2}(5.477\lambda)(4.472\lambda) = 153.89 = 21.87 dB$

c. $e_r e_{cd} \varepsilon_{ap} = 1 \cdot 1 \cdot \varepsilon_{ap} = \frac{1}{2}, \quad \varepsilon_{ap} = \frac{1}{2} = 50\%$

d. $A_{em} = \frac{\lambda^2}{4\pi} G_o = \frac{3^2}{4\pi}(153.89) = 110.2156\ cm^2 = 110.2156 \times 10^{-4}\ m^2$

$P_{rec} = W^i A_{em} = 10 \times 10^{-6} \times 110.2156 \times 10^{-4} = 1{,}102.156 \times 10^{-10} = 11.02156 \times 10^{-8}$

$P_{rec} = 11.02156 \times 10^{-8} = 0.1102156\ \mu\ Watts$

$$a_1 = \frac{G_0}{2\pi}\sqrt{\frac{3}{2\pi\chi}}\,\lambda = \frac{50.7}{2\pi}\sqrt{\frac{3}{2\pi(2.96795)}}\,\lambda = 3.23646\lambda = 8.8268\text{ cm} = 3.475''$$

$$b_1 = \sqrt{2\chi}\,\lambda = \sqrt{2(2.96795)}\,\lambda = 2.43637\lambda = 6.6447\text{ cm} = 2.616''$$

$$P_e = (b_1 - b)\left[\left(\frac{\rho_e}{b_1}\right)^2 - \frac{1}{4}\right]^{1/2} = 6.25263\text{ cm} = 2.46167''$$

$$P_h = (a_1 - a)\left[\left(\frac{\rho_h}{a_1}\right)^2 - \frac{1}{4}\right]^{1/2} = 6.25269\text{ cm} = 2.46169''$$

$$\Rightarrow \rho_e \doteq \rho_h \simeq 6.2526\text{ cm} \simeq 2.4617''$$

13.16. A standard-gain X-band (8.2–12.4 GHz) pyramidal horn has dimensions of $\rho_1 \simeq 13.5$ in. (34.29 cm), $\rho_2 \simeq 14.2$ in. (36.07 cm), $a_1 = 7.65$ in. (19.43 cm), $b_1 = 5.65$ in. (14.35 cm), $a = 0.9$ in. (2.286 cm), and $b = 0.4$ in. (1.016 cm).

(a) Check to see if such a horn can be constructed physically.

13-15

$\rho_1 = 13.5'' = 34.49$ cm

$\rho_2 = 14.2'' = 36.07$ cm

$a_1 = 7.65'' = 19.43$ cm

$b_1 = 5.65'' = 14.35$ cm

$\Rightarrow$ $\rho_e = [\rho_1^2 + (b_1/2)^2]^{1/2} = 13.7924'' = 35.0327$ cm

$\rho_h = [\rho_1^2 + (a_1/2)^2]^{1/2} = 14.7061'' = 37.3536$ cm

$a = 0.9'' = 2.286$ cm

$b = 0.4'' = 1.016$ cm

a. $P_e = (b_1 - b)\sqrt{\left(\frac{\rho_e}{b_1}\right)^2 - \frac{1}{4}} = (5.65 - 0.4)\sqrt{\left(\frac{13.7924}{5.65}\right)^2 - \frac{1}{4}} = 12.544'' = 31.862$ cm

$P_h = (a_1 - a)\sqrt{\left(\frac{\rho_h}{a_1}\right)^2 - \frac{1}{4}} = (7.65 - 0.9)\sqrt{\left(\frac{14.7061}{7.65}\right)^2 - \frac{1}{4}} = 12.529'' = 31.8246$ cm

Therefore $P_e \simeq P_h$, and the pyramidal horn is physically realizable